Counting

Example

List all different results you can get from flipping a coin 4 times

Example

How many ways are there to arrange the letters of $GHOT$?

Example

If we put all our 4 coin-flips in order and converted them to binary, what would we see?

Example

If I listed all possible flips of a coin 10 times in alphabetical order, ie

$$HHHHHHHHHH$$
$$HHHHHHHHHT$$
$$HHHHHHHHTH$$
$$\vdots$$
$$HTHTHTHTHT$$
$$\vdots$$

what would appear 10^{th} in this sequence? 100^{th}?

Arrangements and Permutations

Example (a) How many anagrams of $FRIDAY$ are there?

(b) How many words are there using 4 *different* letters of $FRIDAY$?

(c) How many words are there using 4 *not necessarily distinct* letters of $FRIDAY$?

(d) How many **permutations** of $FRIDAY$ have the two vowels together?

Example

- In how many ways can 15 knights sit around a round table?
- In how many ways can 7 different coloured beads be threaded onto a necklace?

Example

In how many ways can eight rooks be placed on a chessboard so that no two are on the same row or column?

Example

How many numbers divisible by 6 can be made by arranging 4 different digits from $1, 2, 3, 4$ and 5

Example

In how many ways can four married couples be arranged around a round table so that every couple sits together?

Example (a) How many permutations are there of $aaabbbbcc$?

(b) What is the coefficient of $a^3b^4c^2$ in the expansion of $(a+b+c)^9$?

Combinations

Example

In how many ways can a quiz team of six people be selected from a group of twelve people?

Example

In how many ways can a set 0 of two people, set $1A$ of fourteen people and set $1B$ of fourteen people be chosen from a group of 30 people?

Example

In the national lottery a player buys a ticket from a machine which lists six out of the numbers 1 to 49 inclusive. A machine selects six balls, without replacement, from a set of forty-nine balls which are numbered 1 to 49,

(a) How many different ways are there of choosing the six balls?

(b) In how many ways can a player match exactly three of the six winning numbers?

(c) In how many ways can a player match three or more of the six winning numbers?

(d) In addition, the machine draws a seventh ball, which is called the bonus ball. In how many ways can a player match five numbers and the bonus ball?

Example

In how many ways can a committee consisting of five women and six men be formed from eight women and ten men?

Example

Prove that

$$\binom{k}{k}+\binom{k+1}{k}+\binom{k+2}{k}+\cdots+\binom{n}{k}=\binom{n+1}{k+1}$$

Example

Suppose that a,b,c,d,e are positive integers. How many solutions to the equation $a+b+c+d+e=14$ are there?

Example

What if a,b,c,d,e are *non-negative* integers. How many solutions to the equation $a+b+c+d+e=14$ are there?

Example (Poker)
What is the probability of getting:

(a) 1 pair
(b) 2 pair
(c) 3 pair
(d) 3 of a kind
(e) 2 sets of 3 of a kind
(f) 4 of a kind
(g) 1 pair and 3 of a kind
(h) 1 pair and 4 of a kind
(i) Straight
(j) Flush
(k) Straight Flush
(l) No pair

In poker with 5 cards and 6 cards

Example

Calculate $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\cdots+\binom{n}{n}$

Example

Calculate $\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\cdots+(-1)^n\binom{n}{n}$

Example

Calculate $1+2+3+\cdots+n$

Example

Calculate $1^2+2^2+3^2+\cdots+n^2$

Partitions

Example

How many ordered partitions of 10 are there? [Ordered partitions are partitions where the order of the summands matters, eg $3 = 3 = 2+1 = 1+2 = 1+1+1$]

1. How many ordered partitions of n are there?
2. How many *terms* are there in the ordered partitions of n?

Example

How many *unordered* partitions of 5 are there?

Example

Show the number of partitions of n into at most k parts is the same as the number of partitions of $n+k$ into exactly k parts

Example

Show that the number of partitions of n into self-conjugate parts is equal to the number of partitions with *distinct* odd parts

Example

Show that the number of partitions of n into even parts is equal to the number of partitions of n in which each part appears an even number of times

Example

Show that the number of partitions of n into an even number of parts parts minus the number of partitions of n into an odd number of parts parts can be $-1, 0, 1$